RESEARCH ARTICLE

Constrained Probabilistic Multi-objective Optimization of Shot Peening Process

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Shot peening is a cold working process used for improving the fatigue strength of metallic components. An optimum set of peening parameters must increase the residual compressive stress (RCS), but reduce the surface roughness and cold work for improving the fatigue strength. The optimization is made robust to avoid any infeasible solution that may arise out of random variation of input variables. The current study uses the well-known Design and Analysis of Computer Experiments (DACE) methodology for optimization which is better than the conventional Design of experiments (DoE) approach. It employs finite element method (FEM) based unit cell approach to determine the RCS, surface roughness and cold work of a given material. Radial basis functions (RBF) are used to develop the meta-models. Genetic algorithm (GA) is employed for finding a robust and optimum set of shot peening parameters for a given material. With this approach, the operator will achieve the optimum solution, specified by the designer.

Keywords: Shot peening; Probabilistic methods; Radial basis functions; Design and Analysis of Computer Experiments; Genetic algorithm.

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1. Introduction

Imparting residual compressive stresses on the surfaces through shot peening is done to improve the fatigue life of metallic components. The designer’s objective will be to optimize the peening process to obtain maximum fatigue life for the component. To accomplish this objective, the right set of input parameters must be chosen and the optimum values must be obtained by the designer. The operator controls these parameters such that the optimum material responses are within specification limits and the process is repeatable. If the optimum design occurs at the edge/corner of the design space, the design margin may not exist due to random variation of input variables. Therefore, the process needs to be robust enough to accommodate such randomness of input parameters.

DoE studies, both experimental and theoretical, have been employed for optimizing the responses of shot peening process. Figure 1 shows the methodology of DoE in a schematic way. Amongst the optimization studies of shot peening process, only a few are mentioned here due to brevity of space. The optimum values of shot peening on the case carburized 17CrNiMo6 steel are determined experimentally in terms of input variables such as nozzle diameter, air-pressure, the distance between the nozzle and the part, the angle and the mass flow-rate using conventional DoE based approach from Minitab software (Petit-Renaud 2002). Taguchi technique is also used to optimize the shot peening parameters by George et al. (2004). Tufft (1999) has concluded that velocity is the key parameter influencing the intensity. The shot hardness and exposure time are found to have a greater influence over the RCS, while the stand-off distance and the shot diameter have lesser influence (Seno et al. 1990). Peenstress is a software developed by Metal Improvement Company that helps to choose the correct process variables and it predicts the velocity and the RCS (Wandell 1997, Guernic and Eckersley 1996) for a given shot, target material and intensity. The software has a library of materials and geometries to choose from. Baragetti and Terranova (2000) have performed numerical DoE simulations and have identified a non-dimensional parameter (the ratio of RCS magnitude to yield point) to express the shot peening conditions that can be applied to different materials with different treatments. The improvement in alternating bending strength is expressed as a superposition of changes in roughness depth, RCS and hardening (Franz and Olbricht 1987). Vahratian and Garibay (1993) have brought out the need for making the process statistically capable and have performed experiments to optimize the process in an automotive case study.

These studies point to similar, but not the same set of input and output parameters. Secondly, in the classical optimization process, the commonly employed polynomial regression can capture only low-level non-linearity. Also, the gradient-based optimization is likely to capture only local minima or maxima as against the global values. Taguchi techniques have higher noise to signal ratios. A better optimization process, thus, has to be employed to overcome the deficiencies of conventional DoE based approach. As mentioned earlier, probabilistic optimization is also necessary to make the process robust by accounting for random variation of the input parameters, such as shot size, velocity etc.

In the current study, the design space is chosen based on the Almen intensity used in a typical peening operation. The important input parameters are first identified based on the single shot simulation results and the output parameters are selected based on the existing literature. The optimization is multi-objective as both favorable and detrimental response parameters are optimized. Meta-models are developed using the radial basis functions. These models represent the non-linear behavior better than quadratic or cubic polynomials. The genetic algorithm is used to optimize the input parameters based
Figure 1. In this design of experiments approach, the control parameters have to be identified from noise parameters. The design limits are determined for the input parameters. The response surfaces are developed by numerical simulations or experiments using different high-low combinations of input parameters. If non-linear behavior is expected, mid-points are also considered. Optimum values are derived using the gradient-based techniques from the response surfaces.

on the selected response parameters. GA can find the global maxima and minima better than gradient based methods. The input variables may be continuous (e.g. velocity, angle etc) or discrete (e.g. shot radius). The optimized results are then analyzed for probabilistic defects due to uncertainties in input parameters. Then probabilistic optimization is performed to make the optimum values robust.

In this paper, the theoretical aspects of the optimization techniques, that have been used, are explained first. A case study involving Inco718 material is presented. The methodology is, however, applicable to any material. Global sensitivity analysis is performed by single shot simulations. Subsequently, the study describes how the peening parameters are optimized through a constrained multi-objective function with continuous and discrete variables. Probabilistic analysis is then performed to determine whether the variation in input parameters has any sensitivity on the results. The GE in-house software, PEZ (Perl/Eazy Opt) is used in the study.

2. Theoretical aspects of optimization

First a global sensitivity analysis is performed to select the important input parameters. This is accomplished by the FEM based unit-cell approach. The sampling points are then used for building meta-models using radial basis functions. This is followed by deterministic optimization using the genetic algorithm. Uncertainty analysis is then carried out to identify probability of defect from the deterministic optimization. Finally, probabilistic optimization is carried out to reduce the defect and improve the optimum to be more robust. The overall process is explained in Figure 2.
2.1. **Global sensitivity analysis**

The first step is to identify of the input parameters through sensitivity analysis. To accomplish the selection of input parameters, single shot simulations are carried out on a unit cell by varying one parameter at a time. The input parameters that have a high impact on RCS are selected. The response parameters must include both beneficial and detrimental parameters.

2.2. **Sampling through Latin Hypercube**

In typical DoE problems, the limits of design variables are chosen to evaluate the response functions. To capture non-linearity, techniques like central composite design is chosen where few points within the space are also added. However, in Latin Hypercube models,
the design space is divided into \('n \times n'\) squares and one value is chosen for a row and one for a column (See Figure 3) and the selection of the row/column is entirely random. Thus a \('n \times n'\) matrix will have \(n\) sample points. The selection can further be improved by optimizing the values using the optimized Latin Hypercube (OLHC) method, where the randomly chosen sampling points are optimized to have a better distribution in the design space. More information on this can be found in literature such as Ref. (Ramnath and Wiggs 2006).

2.3. **Meta model**

The meta-model is developed to replace expensive analyses or experimentation. It is obtained through a set of Gaussian radial basis functions (RBF). This technique does not assume any specific form, such as a polynomial for the overall approximation function. The RBFs have three layers: input, output and hidden layer (Figure 4). The input parameters of the array \(X\) constitute the input layer; the response parameters form the output layer; and the hidden layer of basis functions, called neurons, calculates the distance between the each input value and the center of that neuron. The basis function is applied to this distance to form the output and is often assumed to be Gaussian. The output is assumed to be a linear superposition of the basis functions.

2.4. **Uncertainty analysis**

To perform uncertainty analysis, the mean \((\mu)\), the standard deviation \((\sigma)\) of the input variables as well as the specification limits (USL, LSL) for response parameters need to be specified. The responses must lie within the LSL and USL limits even with input parameter random variations. The \(Z\) (number of times \(\sigma\)) is calculated as below:

\[
Z_{LSL} = \frac{\mu - LSL}{\sigma}
\]
Similarly, $Z_{USL}$ can be defined with respect to USL. The probability of defect, $p(d)$ is evaluated as the probability that a value will be below (or above) the specification limit, LSL (or USL) respectively. For example,

$$p(d)_{LSL} = 1 - \text{cumulative probability corresponding to } Z_{LSL}$$

(2)

The $p(d)_{USL}$ can be defined the same way with respect to $Z_{USL}$. The total probability of defect, $p(d)_{total}$ is the sum of $p(d)_{LSL}$ and $p(d)_{USL}$.

2.5. **Optimization**

The optimization is carried out through genetic algorithm, which revolves around the genetic reproduction process and *survival of the fittest* strategies. GA differs from classical optimization techniques in that, GA uses a population of points for solution update during every iteration, whereas classical methods use a single solution update. Moreover, classical searches use the deterministic transition rule to move from point to point, but GA uses a probabilistic rule for generating new points in exploring the design space. GA works well for mixed (combination of continuous and discrete parameter) problems. GA then creates a population of solution and selects the best ones using selection, crossover and mutation. $K_t$

2.5.1. **Single objective optimization using genetic algorithm**

A nonlinear constrained optimization, in general, can be mathematically expressed as:

$$\text{Maximize/Minimize } : F(X) \quad \text{(objective function)}$$

Subject to: $g_j(X) \leq 0 \quad j = 1, m$ (inequality constraints),
$h_k(X) = 0 \quad k = 1, n$ (equality constraints),
$X_l^i \leq X_i \leq X_u^i$ (side constraints),
where $[X] = X_1, X_2, ..., X_n$ (design variables).

2.5.2. **Multi objective optimization using genetic algorithm**

The multi-objective optimization is performed by using the Pareto technique. A set of non-dominated trade-off solutions are obtained from which the design engineer can make choices. This set of solutions which are non-competing, is called Pareto front. In this method, a new variable, $\beta$ is added to the existing design variables.

$$X = [X_1, X_2, ..., X_3, \beta]$$

(4)

The problem is redefined as: Minimize $\beta$ subject to $F_k\{X\} \leq \beta \quad k = 1, K$.

2.5.3. **Deterministic Optimization**

In deterministic optimization, the objective function is made to be a combination of the original objective function $f(x)$ and a penalty function $P(x)$:

$$F(x) = f(x) + P(x)$$

(5)

The new objective function $F(x)$ is optimized.
2.5.4. **Probabilistic optimization**

In probabilistic optimization, the formulation of the problem is optimizing a deterministic objective function subjected to probabilistic constraints. These constraints are formulated as probability of defect $p(d)$ on the responses. For each design suggested by GA, an uncertainty quantification through Latin Hypercube sampling is carried out to evaluate the statistical quantities of the responses. Probability of defect of the responses is then computed using the mean ($\mu$), standard deviation ($\sigma$) and specification limits (USL and LSL). GA then finds the optimal solution to the desired $p(d)$ on the responses. Different weight combinations are used to arrive at the Pareto solution.

3. **Case study with Inco718 material optimization**

3.1. **Problem formulation**

In the current study, a 100% coverage is always assumed. Figure 5 shows the intensity (in A type Almen strip) for different velocities and shot diameters. Any material is likely to be shot peened only within a range of Almen intensities. In the current study, the intensity is assumed to be between 0.127 mmA to 0.381 mmA. From the chart, it can be seen that velocity has different upper and lower bounds to have the same intensity range for different shot sizes. This makes the problem to be a constrained optimization.
### Table 1. List of parameters considered for selecting critical input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot size</td>
<td>discrete values</td>
<td>Figure 5</td>
</tr>
<tr>
<td>Shot velocity</td>
<td>25 - 100 m/s</td>
<td>Figure 5</td>
</tr>
<tr>
<td>Shot material</td>
<td>Steel</td>
<td>for density</td>
</tr>
<tr>
<td>Shot hardness</td>
<td>Rigid</td>
<td>mathematically simple</td>
</tr>
<tr>
<td>Impact angle</td>
<td>45 to 90°</td>
<td></td>
</tr>
<tr>
<td>Target material</td>
<td>Inco718</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td>0.0 to 0.5</td>
<td></td>
</tr>
</tbody>
</table>

3.2. Identification of input and response parameters

Determining the right parameters will reduce the optimization time and cost. The important control parameters that affect the work hardening and hence the residual compressive stresses are given in Table 1. The shot is assumed to be rigid (with the density of steel). The target material is chosen to be Inco718. Then, shot size, velocity, angle of impact and friction are the remaining important input parameters that need to be considered.

A single shot impact analysis is performed to determine the effects of different peening parameters. The FEM model is shown in Figure 6. ABAQUS/Explicit software is used for finite element simulation, and a unit cell of 1 mm × 1 mm × 1.25 mm is used as the target material. The cell is modeled with C3D8R (hexagonal elements with reduced integration) elements. The entire surface, except the top surface is coated with infinite elements to prevent stress waves from reflecting from side and bottom surfaces. The XY plane coincides with the top surface and the Z direction is normal to the plane.

The target material, Inco718 is assumed to follow isotropic strain-hardening with rate-dependent stress-strain relations. Though combined hardening is ideally to be used, the current study with isotropic hardening has shown that the RCS distribution is conservative and provides better RCS distribution shape. The mechanical properties are given in Table 2. The elastic-plastic stresses and strains are assumed to follow the popular
Table 2. Mechanical properties of Inco718

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Property</th>
<th>Inco718</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hardness (HRC)</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>Yield Strength</td>
<td>1036 MPa</td>
</tr>
<tr>
<td>3</td>
<td>% Elongation</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Ultimate Strength</td>
<td>1240 MPa</td>
</tr>
<tr>
<td>5</td>
<td>Density</td>
<td>8100 kg/m³</td>
</tr>
</tbody>
</table>

Table 3. Johnson-Cook material constants of Inco718

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inco718</td>
<td>1</td>
<td>5720</td>
<td>10</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 7. Residual stress variation due to single shot impact for velocity variation.

Johnson-Cook equation, which is given without temperature effects as:

\[
\sigma = [A + B(\dot{\varepsilon})^n]\left[1 + C\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] 
\]  

(6)

where A, B, C and n are material constants to be determined. \(\sigma\) is the stress to be evaluated, \(\dot{\varepsilon}_0\) is the reference values of strain rate, \(\dot{\varepsilon}\) is the strain rate under consideration. The material constants are given in Table 3. The shot is meshed with hexagonal elements, but by using a rigid constraint all elements are connected to its center.

Each parameter is varied while keeping all other parameters at a specified value. For
example, the velocity is varied between 20 m/s to 100 m/s, keeping the shot radius at 0.1778 mm, impact angle being normal to surface and zero friction. It is found that the shot size, velocity and angle of impact have a significant impact on the RCS values. Hence, these parameters are considered for optimization studies. The results of these

Figure 8. Residual stress variation due to single shot impact for size variation

Figure 9. Residual stress variation due to single shot impact for angle variation
analyses are given in Figures 7, 8 and 9. Friction is not included as its effect is found to be negligible.

The RCS, the plastic strain and the surface roughness are developed during shot peening as material responses. The RCS is assumed to affect the mean stress due to the applied loads and thus it helps to improve the fatigue life. The area under the RCS distribution curve between the surface and the maximum values has been used as a measure of fatigue strength property in Ref. (Xu et al. 1981). In the current study, the entire area under compression in the RCS curve is used as the key response parameter that will be maximized. This will reduce the error between extremely hard and soft materials.

Shot peening deteriorates the surface roughness significantly. Simpson and Probst (1987) have concluded that the optimum intensity range is determined by the peening induced surface damage rather than by RCS. Surface roughness causes an increase in the stress concentration factor, thus reducing the fatigue life. The surface roughness due to shot peening is analyzed as a local increase in the far-field stress (Rodopoulos et al. 2002). The roughness on the top surface is evaluated from the deformation in the vertical direction, $Z_i$:

$$R_a = \frac{1}{L} \sum_{i=1}^{N} Z_i$$  \hspace{1cm} (7)

The roughness is related to the theoretical stress concentration factor $K_t$ by Neuber’s relation (Arola and C.L. Williams 2002),

$$K_t = 1 + n\sqrt{\frac{R_a}{r}}$$  \hspace{1cm} (8)

where $R_a$ is the roughness, $r$ is the root radius of the surface deformation, $n=1$ for shear loading and $n=2$ for tensile loading.

The cold work is responsible for stress relaxation of the RCS. Thus, from the designer’s point of view, the RCS distribution (the area under compression in the RCS curve) has to be maximized, while surface roughness and cold work are to be minimized, resulting in a multi-objective optimization.

### 3.3. Design space sampling

A Latin Hypercube based DoE matrix is evolved with the shot size, velocity and angle as input parameters. As mentioned earlier, shot radius is a discrete variable. However, all the variables are assumed to be continuous including the shot size. This will help in performing the probabilistic design later, where the variations of all input variables including shot size will be considered. The sampling data containing velocity and shot radius used in this study are shown in Figure 10. As the intensity range is constant for all shot sizes, the velocity range decreases for an increase in the shot size.

### 3.4. Meta-model development

In the case study, the multiple shot simulations are carried out using FEM with the unit cell approach by varying the three parameters, viz., size, velocity and angle. For each sampling point, a specific set of shot size, velocity and angle are chosen. A single shot
Figure 10. Sampling data for shot size and velocity showing the constraints. These data are obtained from optimized Latin Hypercube model.

Figure 11. Multiple shot simulation through random process

analysis is performed for each set of sampling points to obtain the size of the indentation. The FEM model is the same as used in single shot analysis used for global sensitivity analysis.

For obtaining the area under the RCS curve, cold work and roughness, multiple impact simulations with random locations are performed. The FE model is the same as before. In this simulation, the shots impact the central area of 0.25 mm × 0.25 mm at the top
Figure 12. Finite element model shown with 50 shots of 0.25 mm radius at 60°. Only a few shots are shown in the figure.

surface of the unit cell. A computer program is used to generate the random locations of the shots that will provide 100% coverage in the specified zone. The process is explained in Figure 11. The mesh information of the impacted surface is obtained first. A random number is generated in the XY plane to depict the center of the shot. As the shot impacts the surface, the ratio of the number of nodes within the indentation to the total number of nodes determines the coverage ratio. The number of affected nodes are evaluated using the dent size from single shot simulation. The process is repeated for each impact and the coverage is progressively calculated. The vertical dimension of the shots is controlled to avoid any interference among them. Depending on the random seed, the number of shots required for 100% coverage will differ. Less number of shots will result in lower RCS area due to less energy transfer, while more number of shots will increase the simulation time. The computer program is run a large number of times from which the mean number of shots and their locations are obtained.

These locations are used to generate the shots in the FE model in the ABAQUS/Explicit program. Thus, in each multi-shot simulation, depending on the indentation size, the number of shots will vary. All the generated shots are oriented along the same inclination and provided with the given initial velocity. Figure 12 depicts the arrangements of the shots in the unit cell based FEM simulations for a sampling point. As the shots impact the top surface, the surface undergoes plastic deformation, which induces the RCS as well as roughness. The RCS is computed at each layer of nodes starting from the the impact zone. The area under the RCS curve is calculated from these values for each multiple shot simulation. The maximum cold work and surface roughness are also recorded for each sampling point.

Radial basis functions are used to create meta-models for the areas under RCS, cold work and roughness separately. These meta-models help to reduce very long FEM simulations that extend for many days depending on the shot size, number of shots needed for 100% coverage, velocity, and angle. Figure 13 shows the area under RCS curve meta-model for 0.1778 mm radius for different velocities and angles. The response parameter has the unit of MPa.mm. Similar meta-models are developed for the shot radii of 0.254, 0.300, 0.3556 and 0.4064mm. The cold work and the roughness are also used to create
Figure 13. The area under compression in RCS curve for 0.1778 mm shot radius. Non-linearity is observed in the lower velocity range.

<table>
<thead>
<tr>
<th>Meta-model</th>
<th>Radius</th>
<th>Angle</th>
<th>Velocity</th>
<th>RCS area</th>
<th>Cold work</th>
<th>Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial</td>
<td>0.254</td>
<td>81.8</td>
<td>90</td>
<td>3326.8</td>
<td>0.947</td>
<td>0.022</td>
</tr>
<tr>
<td>RBF</td>
<td>0.254</td>
<td>80</td>
<td>90</td>
<td>2460.7</td>
<td>0.610</td>
<td>0.015</td>
</tr>
<tr>
<td>FEM</td>
<td>0.254</td>
<td>81.8</td>
<td>90</td>
<td>2427</td>
<td>0.58</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 4. Comparison of meta-model prediction between RBF and polynomial function fits.

meta-models for all shot sizes. Once the meta-models are developed, a few more FEM simulations are carried out for the validation of meta-models. A typical value from meta-models formed through radial basis and polynomial functions is compared with FEM results in Table 4. It can be seen that RBFs provide better prediction than polynomial fit in comparison with FEM results and hence the optimized solution are more accurate. It is to be noted that the polynomial functions used here are cubic in nature and the response surfaces have $R^2 \approx 99\%$ and do not pass through all sampling points. The RBFs on the other hand have $R^2 = 100\%$ and pass through all sampling points.

### 3.5. Deterministic optimization

GA is employed to perform the deterministic optimization by treating the velocity and angle continuous variables for each radius. The meta-models developed before have been used for optimization. Thus, five different optimization runs have been carried out, one
Figure 14. Variation of $K_t$ with roughness from FEM simulations. The data shows non-linear relationship between $K_t$ and the roughness.

for each shot size (0.1778mm, 0.254mm, 0.300mm, 0.3556mm and 0.4064mm radius) and Pareto surfaces are formed for all shot sizes.

3.6. Probabilistic analysis

In reality, the three input parameters, shot size, velocity, and impact angle vary in a random manner around the set value. Though the shot radius is discrete, it is also likely to vary in a random manner about that mean value. For example, Vahratian and Garibay (1993) have mentioned ±0.75 psi as the range of variation in pressure. Depending on the nozzle diameter, the shot size and the mass flow rate, the velocity of the shot will change. In the current study, variation of 3 m/s is assumed for velocity, indicating a standard deviation, $\sigma$ of 1 m/s. The ±5% variation is assumed to be 3$\sigma$ variation for shot radius and impact angle.

The variation of $K_t$ with respect to roughness obtained from simulations is plotted in Figure 14. Based on the curve, a value of 15 microns is specified as the upper specification limit for roughness, for a $K_t$ of 2. Cammett et al. (2005) have determined that a cold work of 30% is reached for 100% coverage on Inco718, and this is taken as the mean value and 60% cold work is assumed to be the upper specification limit. Similarly, a lower specification limit of 1300 MPa.mm is chosen for area under RCS curve. From these values, the Z values are calculated. In Table 5, a typical set of deterministic optimum values are shown along with the corresponding input parameters. As can be seen in Table 5, the probability of defect, $p(d)$, reaches 1 for cold work, making it infeasible, as the maximum cold work has exceeded its respective specification limits.

3.7. Probabilistic optimization

A maximum value of 1e-05 is chosen for $p(d)$ and GA is again used to reduce the $p(d)$ below this value. As can be seen in Table 6, $p(d)$ is significantly reduced for cold work values. This implies that the values obtained after this step are robust enough such that the random variations of the input parameters still result in infeasible solutions only for a probability of 1e-5. However, the RCS area has become lower than the value from
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Radius</th>
<th>Velocity</th>
<th>$\theta$</th>
<th>Area</th>
<th>Eq. plastic strain</th>
<th>$R_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>units</td>
<td>mm</td>
<td>m/s</td>
<td>deg.</td>
<td>MPa.mm</td>
<td>-</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.1778</td>
<td>67.61</td>
<td>78.72</td>
<td>1940.26</td>
<td>0.70</td>
<td>10.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.00296</td>
<td>1</td>
<td>1.67</td>
<td>31.434</td>
<td>0.0127</td>
<td>0.228</td>
</tr>
<tr>
<td>USL</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1300</td>
<td>-</td>
<td>6</td>
</tr>
<tr>
<td>LSL</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Max $p(d)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5. Probability of defect for shot radius of 0.1778 mm before probabilistic optimization for one optimized point. Cold work crosses 60% that causes $p(d)$ to reach a value of one for this data point.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Radius</th>
<th>Velocity</th>
<th>$\theta$</th>
<th>Area</th>
<th>Eq. plastic strain</th>
<th>$R_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>units</td>
<td>mm</td>
<td>m/s</td>
<td>deg.</td>
<td>MPa.mm</td>
<td>-</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.1778</td>
<td>52.975</td>
<td>73.99</td>
<td>1570.89</td>
<td>0.502773</td>
<td>7.08</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.00296</td>
<td>1</td>
<td>1.67</td>
<td>23.4733</td>
<td>0.0139</td>
<td>0.218</td>
</tr>
<tr>
<td>USL</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1300</td>
<td>0.6</td>
<td>15</td>
</tr>
<tr>
<td>LSL</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Max $p(d)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>1.29e-12</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6. Probability of defect for shot radius of 0.1778 mm after probabilistic optimization for the same optimized result in the previous table. For this data point, the $p(d)$ has been lowered below 1e-5.

Figure 15. Comparison of optimized values for different shot sizes in deterministic optimization. As the shot size increases, the cold work reduces for the same RCS area. Depending on the weights, a set of input parameters can be chosen.

deterministic optimization.
4. Results and Discussions

In the Figure 15, the optimized values between the area under RCS and equivalent plastic strain are compared after deterministic optimization. The cold work has reached beyond 80% which is way above the USL for cold work. The shot with minimum radius (0.1778 mm) produces the highest cold work for the same RCS area followed by the next shot size (0.254 mm). This is followed by shots with 0.300 mm, 0.3556 mm and 0.4064 mm radii. This provides an opportunity for the designer to choose the highest shot size, with which the smallest feature in the component can be peened. This will result in less cold work for the same RCS area. Similar deductions can be performed for surface roughness.

If the optimized point is close to the constraint space, the robust design will be shifted to a less optimum location in order to make the solution feasible. Figure 16 shows the area
Table 7. Specifications to the designer and the operator

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Mean</th>
<th>Range (±3σ)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Intensity mm</td>
<td>0.2</td>
<td>0.05</td>
<td>Based on minimum part dimension</td>
</tr>
<tr>
<td>Coverage</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shot radius, mm</td>
<td>0.1778</td>
<td>±0.006</td>
<td>Based on minimum part dimension</td>
</tr>
<tr>
<td>Shot velocity, m/s</td>
<td>53</td>
<td>±3</td>
<td>Equivalent pressure or wheel speed</td>
</tr>
<tr>
<td>Impact angle, deg.</td>
<td>74</td>
<td>±5</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusions

The current study describes an optimization process that solves multiple objective functions whose input parameters are constrained. It utilizes the advanced OLHC technique to generate the sampling points. The meta-models are formed by using the radial basis functions which are optimized using GA techniques and the responses are made robust by including the random parametric variation. Thus the maximum area under the RCS curve is obtained while keeping the cold work and roughness below the specification limits. A set of possible solutions are obtained for the given material, Inco718 and different discrete shot sizes. However, this process is equally applicable to all materials.

The relaxation due to cold work and fatigue strength reduction due to surface roughness can be quantified through engineering relations. The input parameters such as standoff distance, pressure or centrifugal wheel speed, mass flow rate etc. can be related to the velocity using known relationships. Thus the designer can arrive at a set of robust optimum values that can be achieved by the operator.

6. Acknowledgments

We like to acknowledge GE Aviation for its support towards this project. We also like to thank Jayavenkateswaran, Shamik Chaudhuri and Vinay Ramnath for their help during the study.
References


