

# ENGINEERING FRACTURE MECHANICS

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- Assignment No.3 Mechanisms of Crack Growth and Fracture
- Assignment No.4 Energy Release Rate, Compliance Approach
- Assignment No.5 Crack-tip Stress Field Equation
- Assignment No.6 Stress Intensity Factor Evaluation
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- Assignment No.11  $J$ -Integral
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#### **Notes**

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