High Schmidt number natural convection boundary layers with blowing

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Abstract

We investigate the effects of uniform wall-normal velocity (V_b) on high Schmidt number ($Sc \simeq 600$) indirect natural convection boundary layers. The study is limited for the negligible inertial effects ($Re = V_b\delta_v/\nu$) and negligible diffusive mass transfer in δ_d. Using integral boundary layer equations, we define a non-dimensional blowing parameter, $S = (Re_L^5/ Gr_L)^{1/8}$ to quantify the strength of normal blowing relative to buoyancy and viscous effects. The analysis is conducted for $0.1<S<0.26$. Blowing increases the velocity boundary layer thickness (δ_v), the species boundary layer thickness (δ_d), the horizontal velocity (u) and the wall shear stress (τ_w). We show that $\delta_d/x = 1.59(Re_x/Gr_x)^{1/4}$ and $u(\eta,x)/V_b = \hat{u}(\eta)\,(Gr_x/Re_x)^{1/4}$. Unlike in shear boundary layers, τ_w increases with increase in blowing parameter and the coefficient of drag, C_D scales as $2.4/Re_L$. The concentration in the species boundary layer averaged over half of the mean plume spacing, $\lambda/2$ has a quadratic dependence on the vertical coordinate.

Keywords: Natural convection, Mass transfer phenomena, Plume dynamics.

Introduction

Wall-normal flow through natural convection boundary layers (NBL), formed on permeable horizontal surfaces, results in non-intuitive behavior such as increase in horizontal velocity and wall-shear stress. The problem has similarity solution only for a specific horizontal distribution of wall-normal velocity as shown by Gill et al [1] and Clarke & Riley [2]. However, Lin & Yu [3] and Puthenveettil & Arakeri [4] obtained solutions for uniform V_b, albeit at Pr < 7. In the present analysis, we study indirect NBL subjected to uniform normal blowing in the high Sc regime; the schematic of the present problem is shown in Fig. (1). We investigate the scaling of the species boundary layer thickness, the horizontal velocity and the shear stress in the range of V_b for which the concentration in the thin species boundary layer can be approximated to be uniform. Also, we derive an averaged concentration profile by averaging the uniform concentration profile used in our analysis over a length $\lambda/2$; λ is the mean plume spacing proposed by Puthenveettil and Arakeri [5].

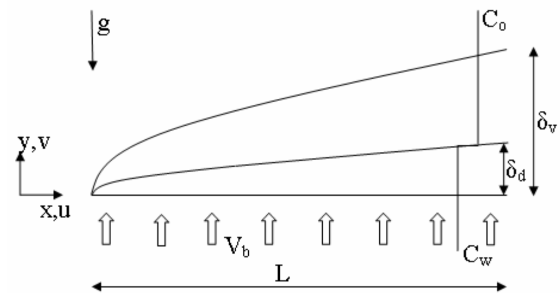

Figure (1): Schematic of the present problem

Formulation

The governing equations for a steady, laminar, indirect natural convection boundary layer are integrated across the boundary layer thickness. A uniform wall-normal velocity, V_b with constant concentration C_w is used as the boundary condition at the wall; the ambient has a concentration of C_o. The momentum and the species conservation equations under uniform concentration approximation,

$$\frac{d}{dx}\int_0^{\delta_v} u^2 dy - \frac{1}{2} g\beta\Delta C \frac{d}{dx}\left(\delta_d{}^2\right) + \nu\left(\frac{\partial u}{\partial y}\right)_{|\,y=0} \quad (1)$$

$$\frac{d}{dx}\int_0^{\delta_d} u\, dy = V_b \quad (2)$$

are non-dimensionalized using the characteristic scales proposed by Puthenveettil [6]. From the non-dimensional equations, we define a blowing parameter $S = (Re_L^5/ Gr_L)^{1/8} = (V_b/V_{ff}^{2/5}V_\nu^{3/5})^{5/8}$ which shows the strength of the blowing relative to the buoyancy and the viscous effects; $Re_L = V_bL/\nu$ is the Reynolds number; $Gr_L = g\beta\Delta CL^3/\nu^2$ is the Grashoff number; $V_{ff} = (g\beta\Delta CL)^{1/2}$ is the free-fall velocity and $V_\nu = \nu/L$ is the viscous velocity. Here, g, L, β, ΔC and ν are the gravitational acceleration, the horizontal length scale, the salinity expansion coefficient, the concentration difference (C_o-C_w) and the kinematic viscosity respectively. Substituting a cubic velocity profile that satisfies the boundary conditions, the equations are solved for the non-dimensional velocity boundary layer thickness and non-dimensional species boundary layer thickness for the parameters shown in Table 1. The analysis is conducted for $0.1<S<0.26$, the lower limit is decided by the uniform concentration approximation while the upper limit is decided by the condition $V_b\delta_v/\nu$ <1 so that the inertial effects are negligible. The parameters shown in Table 1 correspond to the situation where NaCl solution having concentration C_o=10g/l lies over a porous surface with a weak normal flow of pure water having concentration C_w = 0.

Table 1: Parameters used in the calculation corresponding to NaCl in water

L (cm)	ν x10^3 (cm^2/s)	β x 10^4 (l/g)	Gr$_L$ x10^{-5}	D x10^5 (cm^2/s)	ρ (g/l)
1.0	8.93	7.06	0.8685	1.484	10^3

Results and discussion

Fig.(2) shows the dependence of the non-dimensional species boundary layer thickness (δ_d/L) on the blowing parameter S; increase in S results in larger δ_d/L. The additional lighter fluid in the species boundary layer increases the buoyancy force and hence the horizontal motion pressure gradient which is proportional to δ_d.d/dx(δ_d) (Eq.(1)). Since δ_d and d/dx(δ_d) are interdependent, they are seen to increase so as to balance the increase in the motion pressure gradient. The inset of Fig.(2) shows the dependence of δ_d/δ_{dc}(x) on x/L for different blowing parameters. δ_d/δ_{dc}(x) is constant and equal to 1.59 which implies that

$$\frac{\delta_d}{x} = 1.59\left(\frac{\mathrm{Re}_x}{Gr_x}\right)^{1/4} \qquad (3)$$

where, $\delta_{dc}(x) = (\nu V_b x^2/g\beta\Delta C)^{1/4}$. Since δ_d/δ_{dc}(x) is of order one, we propose δ_{dc}(x) to be the appropriate characteristic scale for δ_d implying that $\delta_d \sim x^{1/2}V_b^{1/4}$. Re_x and Gr_x are the Reynolds number and Grashoff number expressed in terms of x. Blowing increases the horizontal velocity and the velocity gradient at the wall resulting in higher wall-shear stress. Fig.(3) shows the distribution of the normalized horizontal velocity u/u_c(x) in the boundary layers at various x/L for a specific value of the blowing parameter and vice-versa, where $u_c(x)=(V_b^3x^2g\beta\Delta C/\nu)^{1/4}$. All the profiles collapse to a single curve of $\hat{u}(\eta)$ implying that

$$\frac{u(\eta,x)}{V_b} = \hat{u}(\eta)\left(\frac{Gr_x}{\mathrm{Re}_x}\right)^{1/4} \qquad (4)$$

where, η is the non dimensional vertical distance and $\hat{u}(\eta)$ is the non-dimensional horizontal velocity which is independent of the blowing parameter and x/L. We hence propose u_c to be the appropriate velocity scale and the above scaling implies that $u(\eta) \sim x^{1/2}V_b^{3/4}$.

Even though the Fig. (2) shows that the species boundary layer is growing monotonically, in real natural convection phenomena, it becomes unstable at a certain boundary layer thickness due to gravitational effect [7]. This instability in the species boundary layer results in the formation of plumes. The horizontal length scale at which this instability occurs is equal to half of the plume spacing. In experiments, the plumes keep forming and merging as they rise so that the spacing between them varies spatially and temporally. Puthenveettil and Arakeri [5] found that the spacing between the plumes is log normal. We approximate this distribution as a regular array with a mean spacing, λ between them. In experiments, as the plumes keep merging, the sensor kept at a location inside the species boundary layer reads an averaged concentration. We expect that the averaged concentration profile in the experiments to be same as that obtained by averaging the uniform concentration profile used in our analysis over a length of λ/2.

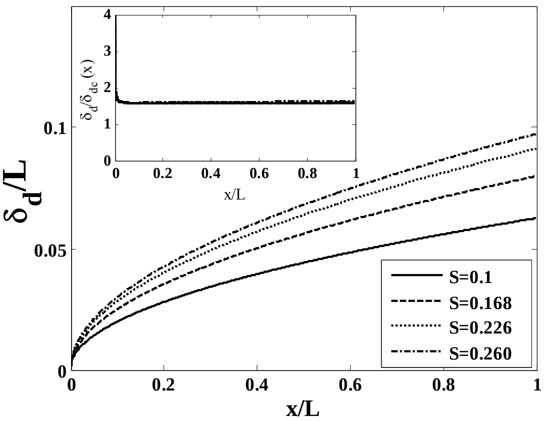

Figure 2: Dependence of δ_d/L and δ_d/δ_{dc}(x) on the blowing parameter

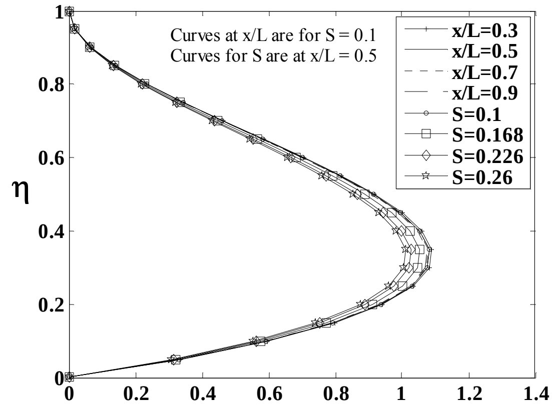

Figure 3: Dependence of u/u_c(x) on the blowing parameter and x/L

The averaged concentration profile is defined as,

$$C'(y) = \frac{1}{\lambda/2}\int_0^{\lambda/2} C(x,y)\,dx \qquad (5)$$

where C(x,y) is the concentration of the species as a function of x and y. λ is the mean plume spacing proposed by Puthenveettil and Arakeri [5] for a normal blowing case which is given by,

$$\lambda=2K^{2/3}Sc^{1/6}(Z_w Z_{V_b})^{1/2} \qquad (6)$$

where, K=0.325, $Z_w = (\nu D/g\beta\Delta C)^{1/3}$ is the near wall length scale in turbulent free convection [8] and $Z_{Vb} = (\nu/V_b)$ is a length scale due to blowing. $\lambda/2$ is the horizontal location at which the species boundary layer becomes unstable when $Gr_{\delta d} \simeq 1$ and

$$\delta_c = K^{1/3}\left(\frac{\nu^2}{g\beta\Delta C}\right)^{1/3} \qquad (7)$$

is the critical boundary layer thickness corresponding to $Gr_{\delta d} \simeq 1$ [7]; $Gr_{\delta d} = (g\beta\Delta C\delta_d^3)/\nu^2$ is the Grashoff number defined in terms of δ_d. The Eq. (5) can be expressed as,

$$C'(y)=\frac{1}{\lambda/2}\left(\int_0^{x_1} C_o dx + \int_{x_1}^{\lambda/2} C_w dx\right) \qquad (8)$$

where, x_1 is the location at any horizontal plane at which the species concentration changes from C_o to C_w which is a function of y as given by the relation,

$$y = 1.59\left(\frac{\nu V_b x_1^2}{g\beta\Delta C}\right)^{1/4} \qquad (9)$$

Eq. (9) is the expression for the edge of the species boundary layer obtained from Eq. (3). Substituting the Eq. (9) in Eq.(8) and non-dimensionalizing results in the non-dimensional averaged concentration,

$$C^* = \frac{C'(y) - C_w}{\Delta C} = 0.837\left(\frac{y}{Z_w}\right)^2 Sc^{-2/3} \qquad (10)$$

Fig. (4) shows the dependence of the non-dimensional averaged concentration on y/Z_w. For $y<\delta_{\lambda/2}$, C^* is proportional to y^2 while it is equal to one for $y \geq \delta_{\lambda/2}$, where $\delta_{\lambda/2}$ is the thickness of the species boundary layer at $\lambda/2$ given by the expression

$$\frac{\delta_{\lambda/2}}{L} = \frac{1.59K^{1/3}}{Gr_L^{1/3}} \qquad (11)$$

$\delta_{\lambda/2}$ is obtained by substituting $\lambda/2$ for x in Eq. (3). C^* is independent of the blowing parameter as the weak blowing doesn't seem to influence the boundary layer stability as it is decided by the gravitational effect. As $\delta_d \propto x^{1/2}V_b^{1/4}$ (Eq.(3)), while $\lambda \propto V_b^{-1/2}$ (Eq.(4)), substituting $\lambda/2$ for x in Eq. (3) makes $\delta_{\lambda/2}$ to be independent of the blowing parameter and it is constant for a given ΔC and the fluid properties.

Conclusions

In the present study, we investigated the effects of uniform blowing on high Sc ($\simeq$ 600) indirect natural convection boundary layers. We proposed a blowing parameter $S = (Re_L^5/ Gr_L)^{1/8}$ which shows the strength of the blowing relative to the buoyancy and the viscous effects. The boundary layer thickness and the velocity profiles at different horizontal positions were obtained for different values of the blowing parameter. Increase in S resulted in larger species boundary layer thickness (Fig.2), higher horizontal velocity and larger wall-shear stress. For $0.1<S<0.26$, δ_d/x scales as $(Re_x/Gr_x)^{1/4}$ and $u(\eta,x)/V_b$ scales as $(Gr_x/Re_x)^{1/4}$ (Fig. 2,3). Below the critical boundary layer thickness, the non-dimensional averaged concentration is observed to have quadratic dependence on the vertical coordinate (Fig.4) and is independent of the blowing parameter.

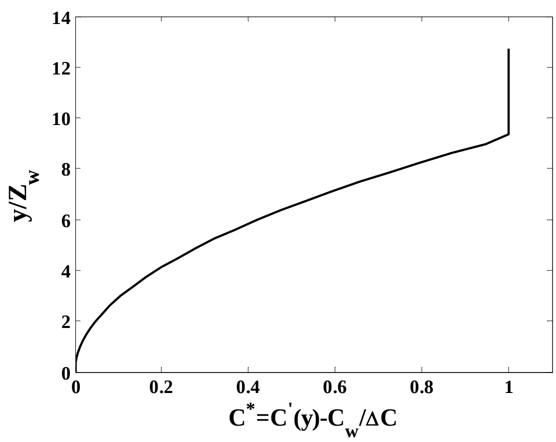

Figure 4: Dependence of the non-dimensional averaged concentration, C^* on y/Z_w.

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