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EFFECT OF WALL-NORMAL FLOW ON HIGH Sc NATURAL CONVECTION BOUNDARY LAYERS

G.Vijaya Rama Reddy*1 and B. A. Puthenveettil*1

*1 Department of Applied Mechanics, IIT Madras, India, gvramareddy@gmail.com

ABSTRACT: We investigate the effects of wall-normal velocity (V_b) on high Schmidt number ($Sc \simeq 600$) natural convection boundary layers formed on permeable horizontal surfaces. Using integral boundary layer equations, we define a blowing parameter, $S = (Re_L^5/ Gr_L)^{1/8}$ to characterize the strength of blowing relative to the buoyancy and the viscous effects. The analysis is performed for $0.1 \leq S \leq 0.26$. The upper limit being given by $Re_\delta = V_b \delta_v / \nu < 1$ so that inertial effects in the boundary layer are small. The lower limit of S ensures that there is negligible diffusive mass transfer in the species boundary layer. As expected, blowing increases the velocity boundary layer thickness and the species boundary layer thickness; the effect is felt more on the species boundary layer thickness. Blowing also increases the horizontal velocity in the boundary layers. We show that the species boundary layer thickness scales as $x(Re_x/Gr_x)^{1/4}$ while the horizontal velocity scales as $V_b\,(Gr_x/Re_x)^{1/4}$.

1. INTRODUCTION

Wall-normal flow through natural convection boundary layers (NBL), formed on permeable horizontal surfaces, results in non-intuitive behaviour such as increase in horizontal velocity and wall-shear stress. The problem has similarity solution only for a specific horizontal distribution of wall-normal velocity (V_b) as shown by Gill et al. [2] and Clarke & Riley [1]. Lin & Yu [3] and Puthenveettil & Arakeri [4] respectively obtained non-similarity and integral solutions for uniform V_b, albeit at Pr < 7. In the present analysis, we study the high Schmidt number (Sc≃600) regime of indirect NBL subjected to uniform normal blowing; the schematic of the present problem is shown in Figure (1). We consider natural convection boundary layers driven by concentration differences caused by a layer of brine over a porous surface with a weak normal flow of pure water from below. Table 1 shows the parameters used in the calculation corresponding to this arrangement. Using integral analysis, we investigate the scaling of the species boundary layer thickness and the horizontal velocity in the range of V_b for which the concentration in the thin species boundary layer can be approximated to be uniform.

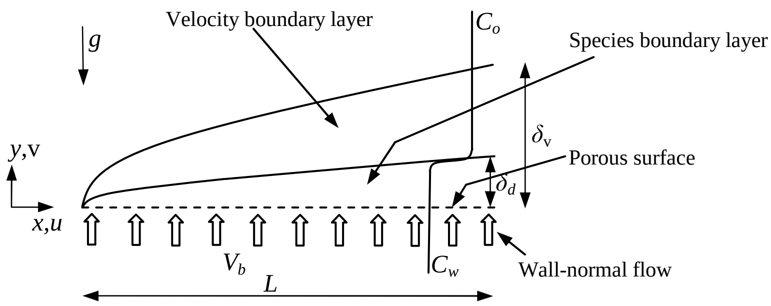

Fig. 1 Schematic of the indirect natural convection boundary layer with wall-normal velocity V_b. Here, C_o is the ambient fluid concentration and C_w is the concentration of the fluid at the wall.

2. FORMULATION

The governing equations for a steady, laminar, indirect natural convection boundary layer are integrated across the boundary layer thickness. A uniform wall-normal velocity V_b with a constant concentration C_w is used as the boundary condition at the wall; the ambient has a concentration of C_o, $C_w < C_o$(See Table 1). In high Sc natural convection boundary layers, as the diffusivity of momentum would be much larger than that of the mass, the thickness of the species boundary layer would be much smaller than that of the velocity boundary layer. A weak blowing would produce a uniform concentration in most of the species

Table 1 Parameters used in the calculation corresponding to NaCl in water. Here, L is the longitudinal length, ν is the kinematic viscosity, β is the salinity expansion coefficient, D is the mass diffusion coefficient and $Gr_L = g\beta\Delta CL^3/\nu^2$ is the Grashoff defined in terms of L, where g is the acceleration due to gravity and $\Delta C = C_o - C_w$ is the concentration difference.

L (cm)	ν (cm^2/s)	β (l/g)	D (cm^2/s)	C_o (g/l)	C_w (g/l)	Gr_L
1.0	8.93 x 10^{-3}	7.06 x 10^{-4}	1.484 x10^{-5}	10	0	86850

boundary layer region. Hence, the concentration in the species boundary layer could be approximated to be uniform and equal to the wall concentration C_w. The integral momentum and the species conservation equations under this approximation are,

$$\frac{d}{dx}\int_0^{\delta_v} u^2 dy - \frac{1}{2} g\beta\Delta C \frac{d}{dx}\left(\delta_d^{\,2}\right) + \nu\left(\frac{\partial u}{\partial y}\right)_{|\,y=0} \tag{1}$$

$$\frac{d}{dx}\int_0^{\delta_d} u\, dy = V_b \tag{2}$$

Here, $\delta_v(x)$ is the velocity boundary layer thickness, $\delta_d(x)$ is the species boundary layer thickness and $u(x,y)$ is the horizontal velocity in the boundary layer. Equation (1) represents the balance of inertial forces, viscous forces and motion pressure gradient in the boundary layers. Equation (2) represents the balance between the convection of species by the horizontal velocity and by the wall-normal velocity. We define the relevant non-dimensional variables as $\hat{x} = x/L$, $\eta = y/\delta_v$, $\hat{\delta}_v = \delta_v/\delta_{vc}$, $\hat{\delta}_d = \delta_d/\delta_{dc}$, $\hat{u} = u/u_c$ and $\hat{v} = v/V_b$. Here,

$$\delta_{dc} \sim \left(\frac{V_b \nu L^2}{g\beta\Delta C}\right)^{1/4}, \qquad \delta_{vc} \sim \left(1+\left(\frac{g\beta\Delta C\nu^3}{V_b^{\,5}L^2}\right)^{1/8}\right)\delta_{dc}, \qquad u_c \sim \left(\frac{V_b^{\,3}L^2 g\beta\Delta C}{\nu}\right)^{1/4} \tag{3}$$

are the characteristic scales proposed by Puthenveettil and Arakeri [5], for the species boundary layer thickness, the velocity boundary layer thickness and the horizontal velocity respectively. These scales are valid for $Re_\delta = V_b\delta_v/\nu < 1$ so that inertial effects are small in the boundary layer; we hence restrict our analysis to weak blowing velocities so that $Re_\delta < 1$. An order of magnitude balance of equation (2) along with the two-dimensional continuity equation, implies that the characteristic scale for the vertical velocity component $v_c \sim V_b$ so that $\hat{V}_b = V_b / v_c = 1$. The non-dimensional form of the integral momentum equation and the integral species equation thus become,

$$\left(\hat{\delta}_v I_{\hat{u}^2}\right)' - \frac{1}{2S(S+1)}\left(\hat{\delta}_d^{\,2}\right)' + \frac{1}{(S+1)^2}\frac{\hat{u}^*_{|\eta=0}}{\hat{\delta}_v} = 0 \tag{4}$$

$$\left(\hat{\delta}_d I_{\hat{u}_{\eta d}}\right)' = 1 \tag{5}$$

Here, $I_{\hat{u}^2} = \int_0^1 \hat{u}(\hat{x},\eta)^2 d\eta$, $I_{\hat{u}_{\eta d}} = \int_0^1 \hat{u}(\hat{x},\eta) d\eta_d$, $\eta_d = \dfrac{y}{\delta_d}$, * denote differentiation with respect to η, ' denote differentiation with respect to $\hat{x}$ and $S = (Re_L^{\,5}/ Gr_L)^{1/8} = (V_b^{\,5}/V_f^{\,2}V_\nu^{\,3})^{1/8}$ is a blowing parameter which characterizes the strength of the blowing relative to the buoyancy and the viscous effects. Here, $Re_L = V_b L/\nu$ is the Reynolds number based on the longitudinal length L, $V_f = (g\beta\Delta CL)^{1/2}$ is the free-fall velocity and $V_\nu = \nu/L$ is the viscous velocity. Substituting a cubic velocity profile that satisfies the boundary conditions, the equations are solved for the non-dimensional velocity boundary layer thickness $\left(\hat{\delta}_v\right)$ and the non-dimensional species boundary layer thickness $\left(\hat{\delta}_d\right)$. The analysis is conducted for $0.1 \le S \le 0.26$, the lower limit is decided by the uniform concentration approximation while the upper limit is decided by the condition $Re_\delta < 1$ so that the inertial effects are negligible.

3. RESULTS AND DISCUSSION

Figure (2) shows the development of the normalized species boundary layer thickness δ_d/L along the normalized horizontal position x/L for different blowing parameters S. Increase in blowing results in larger δ_d/L. The additional lighter fluid in the species boundary layer increases the buoyancy force and hence the horizontal motion pressure gradient which is proportional to $\delta_d \frac{d\delta_d}{dx}$ (eq.(1)). Since δ_d and $\frac{d\delta_d}{dx}$ are interdependent, they are seen to increase so as to balance the increase in the motion pressure gradient. The inset of Figure (2) shows the dependence of δ_d/δ_{dcx} on x/L for different blowing parameters, where $\delta_{dcx} = (\nu V_b x^2/g\beta\Delta C)^{1/4}$. $\delta_d/\delta_{dcx} \simeq 1.59$, which implies that

$$\frac{\delta_d}{x} \simeq 1.59\left(\frac{\mathrm{Re}_x}{Gr_x}\right)^{1/4}, \tag{6}$$

where $Re_x = V_b x/\nu$ and $Gr_x = (g\beta\Delta C x^3/\nu^2)$. Since δ_d/δ_{dcx} is of order one, we propose δ_{dc} (eq.(3)) to be the appropriate characteristic scale for the species boundary layer thickness δ_d implying that $\delta_d \sim x^{1/2} V_b^{1/4}$.

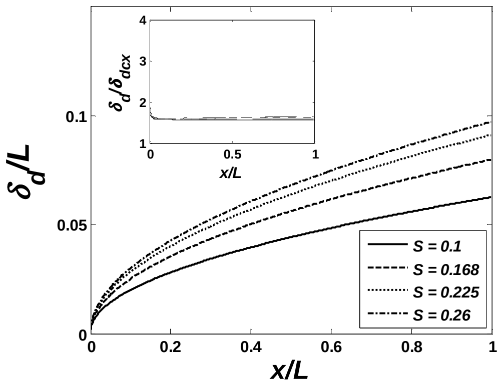

Fig. 2 Variation of the normalized species boundary layer thickness (δ_d/L) with the blowing parameter (S) and the normalized horizontal position (x/L). The inset shows that δ_d/δ_{dcx}, where $\delta_{dcx} = (\nu V_b x^2/g\beta\Delta C)^{1/4}$, is independent of the blowing parameter (S) and the normalized horizontal position (x/L).

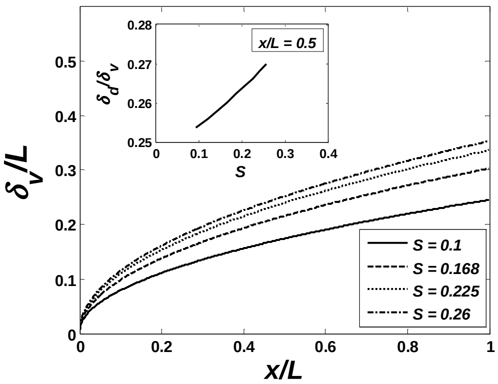

Fig. 3 Variation of the normalized velocity boundary layer thickness (δ_v/L) with the blowing parameter (S) and the normalized horizontal position (x/L). The inset shows the variation of the ratio δ_d/δ_v with the blowing parameter (S) at the normalized horizontal position $x/L = 0.5$

Figure (3) shows the effect of blowing on the normalized velocity boundary layer thickness δ_v/L. Blowing increases the velocity boundary layer thickness. In equation (1), the increase in motion pressure gradient due to blowing should be balanced by the corresponding increase in the inertial and viscous terms. The increase in velocity gradient at the wall seems to be less so that equation (1) may be balanced only if the integration in the first term of equation (1) is performed to higher values of the velocity boundary layer thickness. Hence, blowing results in larger velocity boundary layer thickness. The inset of Figure (3) shows the dependence of the ratio δ_d/δ_v on the blowing parameter S at x/L=0.5. As expected, $\frac{\delta_d}{\delta_v} = \frac{\hat{\delta}_d}{\hat{\delta}_v}\frac{S}{S+1} < 1$ for high Sc boundary layers. The lower species boundary layer thickness is also noticeable from figure (2) compared to the velocity boundary layer thickness in figure (3). As δ_d/δ_v increases with increase in the blowing parameter, the effect of wall normal velocity is felt much more on the species boundary layer than on the velocity boundary layer.

Figure (4) shows the effect of blowing on the horizontal velocity distribution in the boundary layer at x/L = 0.5. Blowing increases the velocity gradient at the wall as well as the velocity inside the boundary layer. Increase in the velocity boundary layer thickness, noted in figure (3), can also be observed in Figure (4). The rise in velocity due to blowing is the result of increase in inertial effects (1st term in eq.

(1)) due to the increase in the motion pressure gradient with blowing. Figure (5) shows the distribution of the normalized horizontal velocity u/u_{cx}, where $u_{cx} = (V_b^3 x^2 g\beta\Delta C/\nu)^{1/4}$, in the boundary layers at various *x/L* for a specific value of *S* and vice-versa. All the profiles collapse to a single curve of $\hat{u}(\eta)$ implying that

$$\frac{u(x,y)}{V_b} = \hat{u}(\eta)\left(\frac{Gr_x}{\mathrm{Re}_x}\right)^{1/4}, \quad (7)$$

where $\hat{u}(\eta) = k(1-\eta)^2\eta$ with *k* as a constant, is independent of *S* and *x/L*. We hence propose u_c (eq.(3)) to be the appropriate velocity scale for *u*(*x*,*y*) and the above scaling implies that $u(x,y) \sim x^{1/2} V_b^{3/4}$.

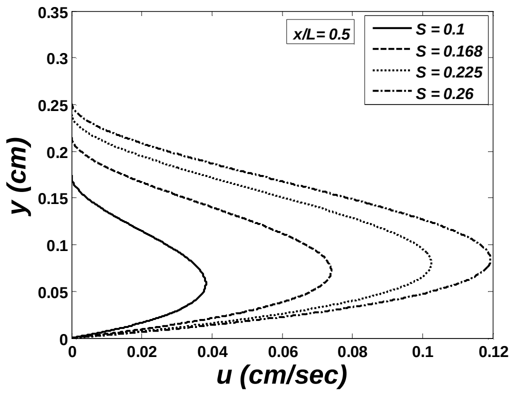

Fig. 4 Effect of blowing on the horizontal velocity distribution at the normalized horizontal position *x/L*=0.5.

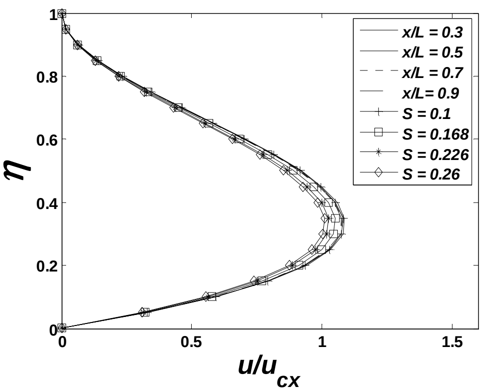

Fig.5 Dependence of u/u_{cx} on the blowing parameter *S* and the normalized horizontal position *x/L*. The curves drawn at different *x/L* are for *S*=0.1 and the curves drawn for different *S* are at *x/L*=0.5.

4. CONCLUSIONS

In the present study, we investigated the effects of uniform wall-normal blowing on high Sc natural convection boundary layers formed on a horizontal permeable surface. We proposed a blowing parameter $S = (Re_L^5/ Gr_L)^{1/8}$ to characterize the strength of the blowing relative to the buoyancy and the viscous effects. The effects of varying S between 0.1 and 0.26 were investigated so that the inertial effects remain small and the species boundary layer could be considered to be of uniform concentration. Increase in *S* resulted in larger boundary layer thickness (Fig.2,3); the effect of blowing was more on the species boundary layer than on the velocity boundary layer (inset of Fig.3). Larger blowing velocities resulted in higher horizontal velocities (Fig.4). $\delta_{dc} = (V_b \nu L^2/g\beta\Delta C)^{1/4}$ was shown to be the appropriate characteristic scale for the species boundary layer thickness (inset of Fig.2); δ_d/x scales as $(Re_x/Gr_x)^{1/4}$ (eq.(6)). The dependence of horizontal velocity was shown to be as $u(x, y)/V_b \sim (Gr_x/Re_x)^{1/4}$ (eq.(7)). Experiments are being conducted to compare the results of the present analysis.

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