Spatial droplet velocity and size profiles in effervescent atomizer-produced sprays

Mahesh V. Panchagnula, Paul E. Sojka*

Thermal Sciences and Propulsion Center, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, 1003USA

Received 10 November 1997; received in revised form 27 October 1998

Abstract

Effervescent atomizer produced sprays were studied for application to rotary kiln incinerators. The sprays were characterized by their number averaged droplet velocity profiles, Sauter mean diameter (SMD) profiles, and size-velocity correlations. Spray mass flow rates ranged from 30 to 120 g/s. Air-liquid-ratios by mass (ALRs) were between 2% and 10%. Data were obtained at seven to ten radial positions across the diameter of the spray at three separate axial locations, all in the far-field (or dilute region) of the spray. The number averaged droplet velocity profiles were bell-shaped across any diameter, irrespective of ALR, liquid mass flow rate, or axial location. Velocity magnitudes were found to increase with an increase in either liquid mass flow rate or ALR, and to decrease with an increase in axial distance from the atomizer exit. SMD was found to be nearly constant across any spray diameter. On the basis of velocity-size-number flux contour plots, it was determined that the droplet size-velocity correlation is minimal. This observation, coupled with the minimal variation in SMD across a diameter, indicates that the spray can be modeled as a variable-density single-phase jet. A model was developed, based on previous gas-phase studies, to explain the development of the drop velocity profiles in the dilute spray region. Agreement between the measured drop velocities and model predictions was within 30% in all cases. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Effervescent sprays; Velocity-dropsize profiles

Nomenclature

\begin{align*}
\text{ALR} & \quad \text{air liquid ratio by mass, dimensionless} \\
\theta, b, c, d & \quad \text{empirical constants, dimensionless} \\
D & \quad \text{exit orifice diameter, m} \\
J & \quad \text{jet momentum flux at nozzle exit, N} \\
K & \quad \text{empirical constant, dimensionless} \\
\dot{m} & \quad \text{mass flow rate, kg/s} \\
r & \quad \text{radial coordinate, m} \\
Re & \quad \text{Reynolds number, dimensionless} \\
\dot{u} & \quad \text{mean velocity in the spray, m/s} \\
\bar{U} & \quad \text{dimensional group, m/s} \\
U_{\text{max}} & \quad \text{centerline velocity in the spray, m/s} \\
v & \quad \text{velocity, m/s} \\
x & \quad \text{axial distance from the atomizer exit, m} \\
\eta & \quad \text{velocity profile similarity parameter, dimensionless} \\
\Theta & \quad \text{angle, degrees} \\
\rho & \quad \text{density, kg/m}^3 \\
\sigma & \quad \text{spray development empirical parameter, dimensionless} \\
\text{Subscript} \\
e & \quad \text{entrained gas} \\
g & \quad \text{gas} \\
l & \quad \text{liquid} \\
o & \quad \text{exit orifice}
\end{align*}

Greek symbols

\begin{align*}
\alpha & \quad \text{void fraction, dimensionless}
\end{align*}

1. Introduction

Effervescent atomization is a method of spray formation that intimately mixes a non-condensable gas with the liquid to be sprayed. The resulting two-phase mixture passes through an orifice where the rapid expansion of the gas breaks the liquid up into a multitude of drops.

A form of effervescent atomization was reported by
Chawla [1] who noted that it produced sprays with drop sizes independent of atomizer exit orifice diameter and claimed this was because of a choked two-phase flow at the final orifice. This insensitivity of atomizer performance to final orifice diameter suggests effervescent atomizers will be less sensitive to plugging than pressure and conventional twin-fluid nozzles when operating with particulate laden fluids. Chawla [1] also noted that only low fluid velocities are required to produce fine sprays and that the energy required is substantially less than that of a conventional twin-fluid injector. The former suggests effervescent atomizers will not suffer from the severe erosion problems that are common when pressure and twin-fluid injectors are operated on multiphase feedstocks. The latter suggests that as the drop velocities are lower than in air-assist or airblast atomizers, shorter incinerators will be required for complete drop burnout, thereby reducing capital costs.

It is clear that effervescent atomization holds promise in liquid waste incineration applications. The first advantage is its relative insensitivity to viscosity. This is particularly important when spraying waste products, because their rheological properties usually cannot be controlled. The second major advantage is that performance is nearly independent of nozzle exit orifice diameter. This allows the atomizer to have large bore passages which reduce its tendency to plug. The third advantage is that they improve mixing. As opposed to plain orifice or airblast atomizers, effervescent atomizers are characterized by a multiphase flow that begins immediately at the nozzle exit. This results in higher turbulence levels and improved mixing, which leads to faster burnout. (Typical gas-phase turbulence levels in effervescent sprays are about 10% higher than the highest measured turbulence in the single phase gas jets of Wygnanski and Fiedler. [2]) Faster burnout will yield equivalent destruction levels in shorter lengths, thus allowing incinerator size to be reduced and again reducing capital costs.

Effervescent atomizer based liquid waste incinerators can also employ pilot gaseous fuels as the atomizing ‘air’. The results of Lund et al. [3] have shown that the mean drop size in effervescent sprays increases slightly with an increase in atomizing gas molecular weight. Hence, air, which has been used as the atomizing gas in all previous studies, can be replaced by a pilot fuel like natural gas with a resulting decrease in droplet size. This would improve the performance of the atomizer and at the same time enhance combustion in the incinerator.

A number of studies at Purdue have been performed to investigate effervescent atomizer performance. Lefebvre et al. [4] noted that drop size was a strong function of fluid delivery pressure, but a weak function of atomizer exit orifice diameter. Wang et al. [5] concluded that the sprays formed had SMDs nearly independent of aerator geometry. Roesler and Lefebvre [6] demonstrated that the porosity of the aerator tube had little effect on SMD when pore size was varied between 20 and 120 μm.

Effervescent sprays of non-Newtonian fluids have been the subject of study because of their applications in consumer products, painting/coating, materials processing, and combustion processes. Buckner and Sojka [7] noted that sub-25 μm SMD sprays could be produced using ALRs below 15% for Newtonian fluids with viscosities ranging from 300 to 1000 mPa-s. They also found that fluid rheology had little or no effect on atomization quality, but that polymer addition was detrimental. Air–liquid-ratio (by mass) and nozzle pressure were found to have the strongest effect on SMD. Jardine [8] was able to produce sub-50 μm SMD sprays for mass flow rates up to 940 g/s and ALRs up to 30%, and observed that drop size was weakly related to atomizer diameter and to shear thinning rheology.

Although each of the previous investigations has helped reveal the potential of effervescent atomization, they have all focused on drop size (i.e., on SMD). This approach has left many issues unaddressed.

One important issue that has not been addressed is the spatial variation of drop velocity. Lund et al. [9] did make some number averaged droplet velocity measurements in effervescent atomizer-produced sprays, but all of their data were obtained at mass flow rates of about 1 g/s for relatively low viscosity fluids (< 100 mPa-s). This gap in the literature is troublesome because droplet velocity is a key input to the design of liquid waste incinerators as it has a direct impact on the length required for complete droplet combustion. Unfortunately, there are no correlations available to predict droplet velocity distribution in effervescent sprays.

This study was performed to address the deficiencies of previous investigations into effervescent atomizer performance. Application to liquid waste incinerators was of particular interest, although results are applicable to other liquid combustion processes. Three goals were set:

1. To demonstrate that an effervescent atomizer can produce sub-75 μm drops at high mass flow rates (up to 120 g/s) and at high liquid viscosities (up to 900 mPa-s). These mass flowrates were of interest as they correspond to firing rates of approximately 0.5–2.5 MW, for a single atomizer spraying a fluid whose heating value is 20,000 kJ/kg. These viscosities are of interest as they represent a reasonable upper limit for incinerator feedstocks.

2. To characterize atomizer performance in terms of number averaged droplet velocity, SMD, and the velocity-size-number flux correlation, all with respect to variations in atomizer operating conditions, viz. ALR and liquid mass flow rate. Number averaged velocity and SMD are of interest because of their direct relationship to incinerator length. The velocity-size-number flux correlation is important because the sprays can be modeled as variable-density single-phase jets if there is no correlation between velocity, size and number flux.

3. Most importantly, to develop a model that will predict
the spatial evolution of the droplet velocity profile in the dilute, or far-field, region of the spray. This model can then be used by designer engineers to accurately specify incinerator length.

A Phase/Doppler Particle Analyzer (P/DPA) was used to measure drop velocities and diameters. Goals 1 and 2 were achieved in the experimental portion of this program. A variable-density single-phase jet model was then developed to predict the far-field spatial development of the spray velocity profile.

2. Background

The only measurements of droplet velocity in effervescent sprays were performed by Lund [10]. Similar information for high mass flow rate or high viscosity effervescent sprays is absent from the literature. Consequently, the general features of previous studies on velocity profiles in related two-phase jets will be reviewed.

It is noted here that effervescent jets differ from other two-phase jets in two key respects. First, although particle laden jets are two-phase at the atomizer exit, there is no velocity slip between the two phases at that point; effervescent sprays are characterized by an interphase velocity slip at the atomizer exit. Secondly, liquid sprays differ from effervescent sprays in that liquid sprays are not two-phase at the nozzle exit, but only develop into a two-phase jet further downstream. Therefore, the best that can be expected from the two-phase jet literature is general guidelines for phenomena that occur in all cases.

A large number of articles discuss the various aspects of two-phase jet behavior. They can be divided into spray and particle–laden jet studies.

Briffa and Dombrowski [11] studied velocity profiles in flat sprays. Faeth [12] reports that Shearer et al. [13] predicted droplet velocity profiles in pressure atomized sprays using a model based on the LHF approximation. Their sprays indicated self-similarity in velocity profile at $x/D$ values of about 150. However, in some sprays the predicted centerline velocities were 10%–20% lower than the measured values, which were well above experimental uncertainties. This systematic offset in the predicted values of the droplet velocity was attributed to evaporation. The predicted values were then compared with the single-phase measurements of Wygnanski and Fiedler [2], Hetsroni and Sokolov [14], Becker et al. [15], and Shearer and Faeth [16]. The agreement was reasonably good and results also showed that sprays exhibit self-similarity in droplet velocity profiles, like free gas jets.

Ruff et al. [17] measured drop velocities in pressure-atomized sprays operating in three different breakup regimes. Predictions of entrainment were also made using a model developed under the locally homogenous flow (LHF) assumption. Model predictions were compared with experimentally measured mass flow rates; it was observed that LHF model predictions were substantially higher than experimental values for the first and second breakup regimes, but were somewhat more accurate when predicting entrainment and liquid volume fraction distribution in the third mode. As all effervescent sprays fall into the third mode of breakup, use of the LHF approximation may be justified in predicting droplet velocities for the sprays of interest here.

Mao et al. [18] applied the model of Shearer and Faeth [16] to air- and pressure-atomized n-pentane sprays combusting in air. They used the empirical constants obtained by Shearer and Faeth [16] for the velocity profile and found that the predicted and measured values were in excellent agreement.

The second class of two-phase jet studies that has been reported in the literature describes particle laden jets. Subramanian and Ganesh [19], Shuen et al. [20], Subramanian and Raman [21], and Subramanian and Ganesh [22] studied the structure of two-phase particle–laden jets. They used techniques similar to those used by researchers analyzing liquid sprays and found that particle–laden jets, like liquid sprays, entrain air like a free gas jet after the jet is fully developed.

In summary, there is no direct evidence that effervescent atomizer produced sprays can be modeled as variable-density single-phase jets because of a lack of experimental data. Further, available data for sprays and particle–laden jets cannot be applied directly because effervescent atomizer produced sprays are two-phase and have interphase
velocity slip at the atomizer exit orifice, unlike conventional sprays and particle laden jets. However, there is reason for optimism as both sprays and particle–laden jets have been modeled as variable-density single-phase jets in the past.

3. Experimental apparatus

Fig. 1 illustrates the atomizer employed in this investigation. Liquid is fed into the atomizer at the top and flows downward through a sintered metal porous tube. Air is fed into the annulus surrounding the porous tube and is injected radially into the liquid. The resulting two-phase flow then accelerates through a convergent nozzle to the interchangeable exit orifice, which allows variation in exit orifice diameter. A 3.0 mm diameter exit orifice was employed for the 30 and 60 g/s liquid mass flow rate sprays, while a 4.5 mm diameter exit orifice was used for the 120 g/s sprays.

Atomizer operation requires air and liquid supplies. The liquid supply consists of two spherical reservoirs connected in series. The reservoirs were pressurized using regulated building air in order to feed the liquid to the nozzle. The liquid flow rate was measured using a Micromotion model M40S-SS flow meter. Flow control was achieved using a metering valve and a ball shut-off valve in series. The pressure of the fluid as it enters the atomizer was monitored using a dial indicating gauge.

Atomizing air was supplied by the regulated building air system. A Brooks 1110-24 model rotameter was used to monitor the air flow rate. A dial indicating pressure gauge was used to ensure that the rotameter was operated at the calibrated pressure.

A P/DPA was used to obtain all the drop velocity, size, and number flux data presented here. The instrument was developed by Bachalo and Houser [23]. A schematic of the optical setup is shown in their article. The P/DPA was fitted with a 495 mm transmitting lens and a 300 mm collimating lens. This permitted measurement of drop diameters between 3 and 450 μm (and velocities between ~50 and 110 m/s). The forward mode of scattering at an angle of 30° was chosen.

The P/DPA collects drop size and velocity information from a user selected sample of over 10,000 drops at a particular position in the spray. The number averaged droplet velocity, SMD, and number flux are calculated from that sample.

The P/DPA also outputs information about the size-velocity correlation in the spray. The sample of drops is classified and each drop placed into one of 50 size bins and one of 50 velocity bins. The size-velocity correlation is a plot of the number averaged velocity of drops in a particular size bin versus the bin size.

The single-phase variable-density flow model developed during this study requires knowledge of the spray momentum rate. The spray momentum rate is a sum of the gas and liquid momentum rates exiting the nozzle. It was measured using the probe shown in Fig. 2. This probe is a derivative of that described by Bush et al. [24] and consists of three main parts—the flow diversion system, the load cell assembly, and the electronic data processing system. The flow diversion system is an axisymmetric Plexiglas cone that is mounted on a horizontal plate and is designed to divert an axial flow radially outward. Thus, the axial force acting on the cone is equal to the momentum rate exiting the nozzle.

The load cell used during this experiment is an Omega LCF-10. One end of the load cell is screwed into
that simulate high viscosity industrial wastes. The dynamic viscosity was measured using a rotary viscometer (Haake RV-20) and found to be 900 mPa-s. The surface tension was measured using a Cenco duNuoy unit and determined to be 0.067 kg/s. The density was measured using a centigram laboratory balance and a graduated cylinder. It was calculated to be 1200 kg/m³. All physical property data were obtained at room temperature (nominally 25°C) and the local barometric pressure.

4. Results and discussion

Three categories of results are presented. The set first is plots of number averaged droplet velocity versus radial position for three axial locations (at a single ALR and liquid mass flow rate), plus SMD versus radial position for three axial locations (at a single ALR and liquid mass flow rate). The second set is velocity-size-number flux contour plots. Only two radial positions were chosen for each axial location and choice of ALR and liquid mass flow rate—near the edge of the spray and close to the centerline of the spray. The last set is momentum rate measurements. They are presented as a plot of momentum rate versus ALR for the three mass flow rates investigated.

Figs. 3–5 illustrate how number averaged drop velocity varies with radial position throughout the spray for a variety of operating conditions. Corn syrup mass flow rate was varied from 30 to 120 g/s, the air liquid ratio ranged from 2% to 10%, and the axial distance varied from 30 to 45 cm downstream of the exit orifice. The first point in the radial scan was chosen close to the spray edge such that the P/DPA validation rate was no less than 45%. This was done to increase reliability of the data. Data were then obtained by traversing across a spray diameter at spatial intervals of 10 mm until the validation rate again fell below 45% near the opposite edge. The radial scan was then terminated for that axial position.

Each of the Figs. 3–5 plots includes the mean velocity plus the root mean square velocity, the latter indicated as vertical bars. The velocity profiles are bell-shaped with the maximum velocity occurring along the spray centerline. This symmetry is anticipated because the atomizer is axisymmetric. As expected, the velocities increase with an increase in either ALR or liquid mass flow rate. This increase is ascribed to an increase in the momentum rate of the spray at the atomizer exit.

Fig. 6 is a plot of the centerline velocity versus liquid mass flow rate for the three axial positions and three ALRs. The data were extracted from Figs. 3–5. As expected, centerline velocity decreases with an increase in axial distance from the atomizer exit. The effect of liquid mass flow rate for various ALRs can be observed by comparing Figs. 6a, b and c. A comparison of Figs. 3a, b and c or Figs. 4a, b and c or Figs. 5a, b and c illustrates the effect of ALR.
The gas-phase turbulence levels in the spray were estimated by using the smallest droplet size bin velocities to approximate continuous-phase motion. Calculations were based on the corresponding rms velocity and the mean centerline velocity.

Spray turbulence levels (determined as discussed above) were 10% higher than the highest measured turbulence in single-phase gas jets, as studied by Wygnanski and Fiedler [2]. We therefore conclude that effervescent atomizer produced sprays are highly turbulent. This suggests substantial mixing throughout the spray. One influence of this mixing will be discussed when SMD data are considered.

Figs. 7–9 present SMD versus radial position for three axial locations, two or three ALRs and a single liquid mass.
flow rate. Note that SMD is at or below the target value of 75 \(\mu\text{m}\) in most cases. Also note that the drop size profiles are nearly constant across the spray and exhibit a relatively small variation with liquid mass flow rate and ALR. This is the result of the enhanced mixing discussed in the previous paragraph.

Panchagnula [25] presents contour plots of the 50 x 50 velocity-size-number flux matrix obtained using the P/DPA. The matrix illustrates the qualitative distribution of drops in the spray among the velocity and size bins. They also shed light on how liquid mass flow rate and ALR affect drop velocity and size throughout the spray. The observed trends can be summarized as follows. First, droplet mean velocities are consistently the same for all drop classes. Second, droplet velocity profiles are consistently the same for all drop size classes. These two observations prove that there is no size-velocity correlation for these sprays. Finally, comparison of velocity-size number flux plots for the range of operating conditions and spatial locations considered here supports the variation of the number averaged droplet velocity as observed in Figs. 3–5.

Momentum rate was measured using the momentum rate probe. Fig. 10 presents the variation in momentum rate for three mass flow rates and three ALRs. As expected, the momentum rate increases with an increase in either mass flow rate or ALR.

To summarize, the number averaged velocity profiles at all dilute spray (far-field) axial positions, mass flow rates and ALRs were found to be bell shaped. Further, the SMD was found to be nearly constant across a spray diameter regardless of far-field axial position. In addition, the size-velocity correlation was observed to be negligible. These three observations suggest that the spray drop size distribution can be decoupled from the velocity variation, and the flow treated as a variable-density single-phase jet.

A quantitative model was therefore developed to describe the variation in drop velocity throughout the dilute portion of the spray for a variety of operating conditions. This model is discussed in detail in the following section.

5. Model

The effervescent sprays studied in this work will be modeled as variable-density single-phase turbulent jets. This assumption is justified based on the following experimental findings:

1. Figs. 7–9 show that the radial SMD profiles exhibit little variation across a spray diameter. This is attributed to the intense turbulent mixing in these sprays.

2. The Panchagnula [25] drop size-velocity-number flux plots indicate that there is a negligible size-velocity correlation in the sprays considered, regardless of liquid mass flow rate, ALR, radial position, or axial location.

According to White [26], turbulent jets become fully developed and self-similar at about \(x/D = 70\), where \(x\) is the axial distance from the atomizer exit and \(D\) is the atomizer exit orifice diameter. Benatt and Eisenklam [27] reported full development of spray velocity profiles for \(x/D > 100\). As we are specifically concerned with only
the dilute spray region, the smallest $x/D$ considered in this work is about 100. As a result, we expect the mean velocity profile in the sprays investigated here to be self preserving. Spray properties can therefore be calculated as mass weighted averages of the properties of the liquid and gas phases.

The conclusion that effervescent atomizer produced sprays exhibit self-similar mean velocity profiles is supported by the data in Fig. 11. This data was extracted from Figs. 3–5 and illustrates the variation in centerline velocity with the reciprocal of the dimensionless axial location, i.e., $D/x$. For a jet having a self-similar mean velocity profile, we expect the centerline velocity to show a linear variation with $D/x$. Fig. 11 exhibits this behavior for all liquid mass flow rates and ALRs investigated during this study. As the centerline velocity is a linear function of $D/x$ at all conditions of liquid mass flow rate or ALR, self-similar mean velocity behavior is possible.

The first task during model development was to identify a self-preserving turbulent jet velocity profile equation and fit it to the experimental data to obtain a two-parameter description of the velocity distribution. Abramovich [28] and White [26] list various solutions to the turbulent axisymmetric jet equations that have been obtained by various investigators. According to White [26], Gortler [29] solved the boundary layer approximation to the exact
Navier–Stokes equations with the appropriate boundary conditions to yield the following similarity solution for the velocity profile

\[ \bar{u} = U_{\text{max}} \text{sech}^2(\eta), \]  

where,

\[ \eta = \sigma \frac{r}{x}, \]  

and

\[ U_{\text{max}} = b \sqrt{\frac{J}{\rho_c x}} \]  

Here \( \bar{u} \) is the mean axial velocity at a point in the jet, \( U_{\text{max}} \) is the maximum centerline axial velocity, \( J \) is the jet momentum rate, \( \rho_c \) is the entrained gas density, \( \sigma \) and \( b \) are empirical constants, \( r \) is the radial distance measured from the centerline of the jet, and \( x \) is the axial position measured downstream of the jet exit orifice.

White [26] compared predictions based on this equation to the experimentally measured turbulent jet velocity profile data of Wygnanski and Fiedler [2] and found excellent agreement. This equation was therefore chosen for the spray velocity profiles sprays described here. The two empirical constants involved in this equation, \( b \) and \( \sigma \) can be calculated from experimentally measured velocities, and their variation with experimental parameters like liquid mass flow rate and ALR.

The second task was to accurately identify the centerline of the spray. The centerline of the spray was chosen as the point where the velocity would reach a maximum if infinitesimal resolution experimental data were available. As infinitesimal resolution velocity data were not available, a fitting procedure was employed to locate the spray centerline. In principle, any even function can be fit to the data for the purpose of identifying the centerline. A number of approaches were tried, including Gaussian, quadratic, and cubic spline. All yielded unacceptable agreement. In the end, the experimentally measured number averaged droplet velocity versus radial position profile at each axial location was fitted with the exact polynomial passing through all the points. The exact centerline of the spray was located as the point closest to the highest experimentally measured velocity where the derivative of the exact polynomial was zero and the second derivative was negative. The symmetry of the profile was verified by shifting the origin from the edge of the spray to the centerline and calculating the coefficients of the odd powers. The values of these coefficients were all found to be close to zero indicating that the equation is indeed even about the exact centerline. The exact value of the maximum velocity was calculated from this polynomial.

The full width at half maximum (FWHM) was calculated by solving for the difference between the roots of the exact fit polynomial which correspond to \( \bar{u} = U_{\text{max}} \).

The empirical constant \( \sigma \) was determined from the calculated FWHM and the known axial position.

Eq. (1) is the solution for a free jet injected into quiescent air. It needs modification for application to a two-phase effervescent spray. This was accomplished by introducing the two-phase momentum rate for \( J \) and the entrained gas
density \( \rho_e \)

\[ J = \dot{m}_l v_l + \dot{m}_g v_g \]  

(4)

where \( \dot{m}_l \) and \( v_l \) are the liquid-phase mass flow rate and velocity, \( m_g \) and \( v_g \) are the gas-phase mass flow rate and velocity.

A value for \( b \) in Eq. (3) must be determined for the sprays considered here. In order to do that, a two-phase computing equation must be developed for \( b \), analogous to the single-phase version in Eq. (3).

We begin by substituting Eq. (4) into Eq. (3). This results in

\[ U_{\text{max}} = b \sqrt{\frac{\dot{m}_l v_l + \dot{m}_g v_g}{\rho_e}} \frac{1}{x}. \]  

(5)

Dividing and multiplying by \( \dot{m}_l v_l \) in the radical, and substituting ALR for \( \frac{\dot{m}_g}{\dot{m}_l} \) and sr for \( \frac{v_g}{v_l} \) gives

\[ U_{\text{max}} = b \sqrt{\frac{\dot{m}_l v_l (1 + \text{ALR} \cdot \text{sr})}{\rho_e}} \frac{1}{x}. \]  

(6)

From liquid-phase continuity

\[ v_l = \frac{4 \dot{m}_l}{\pi d_0^2 \rho_l (1 - \alpha)}, \]  

(7)

where \( \rho_l \) is the liquid density, \( d_0 \) is the atomizer exit orifice diameter, and \( \alpha \) is the void fraction. \( \alpha \) can be expressed as:

\[ \alpha = \frac{1}{1 + \left( \rho_g \cdot \text{sr} / \rho_l \cdot \text{ALR} \right)} \]  

(8)

using continuity for the liquid and gas phases [30].

Substituting Eqs. (7) and (8) into Eq. (6) and simplifying, one obtains the predictive equation for \( U_{\text{max}} \)

\[ U_{\text{max}} = b \sqrt{\frac{4 \dot{m}_l}{\pi \rho c d_0 x}} \sqrt{\frac{\rho_l (1 + \text{ALR} \cdot \text{sr})}{\rho_e}} \frac{1}{1 + \frac{\rho_g \cdot \text{sr}}{\rho_l \cdot \text{ALR}}}. \]  

(9)

\( b \) can now be calculated from Eq. (9) by using the maximum velocity, computed as indicated earlier, plus the liquid mass flow rate, ALR and sr measured for each spray.

Knowing these two parameters for each velocity profile the two parameter self-similar jet velocity profile given in Eq. (1) can be used to describe the droplet velocity distribution throughout the dilute region of the spray.

The procedure discussed earlier was repeated for all sprays investigated in this work. The two parameter equation was fit to the number averaged velocity profiles from sprays of varying liquid mass flow rate and ALR and for varying axial position in each spray’s far-field.

Fig. 10 gives \( U_{\text{max}} \) (the centerline velocity) versus a group with the dimensions of velocity

\[ U = \frac{\dot{m}_l}{\rho c d_0 x} \]  

(10)

As predicted by Eq. (9), the plot yields a straight line for each of the ALRs with the slope of the straight line now only a function of ALR. The correlation coefficient is no less than 0.95 in any of these cases.

Fig. 13 is a plot of the calculated values of \( b \) versus ALR for all sprays studied. The parameter \( b \) was calculated for each spray as described above and found to scale approximately linearly with ALR, varying by only about 30% throughout the range investigated. The scatter of points about the straight line is random with liquid mass flow rate and hence can be attributed to experimental uncertainty.
We now have an expression to evaluate the centerline velocity in the dilute region of the spray. The other parameter in Eqs. (1) and (2) that needs to be evaluated is $\sigma$. $\sigma$ can be calculated from the FWHM of each velocity profile. This was done for all the velocity profiles at the three axial locations. It was found that $\sigma$ was constant to within 30% in each spray.

In a turbulent gas jet, the full width at half maximum is a linear function of axial position, giving rise to a constant $\sigma$. However, in effervescent sprays this is not true and $\sigma$ was found to vary with axial location in the spray. To model effervescent sprays as variable-density single-phase jets, $\sigma$ was assumed to be constant in a particular spray and the average of the three $\sigma$ values at the three axial positions chosen.

As mentioned in the previous paragraph, $\sigma$ is not constant throughout a spray. It is a function of the cone angle. The cone angle depends on liquid mass flow rate and ALR. In addition, the cone angle is obviously a function of the exit orifice diameter, as it is in a free gas jet. Dimensional analysis of these parameters indicates that $\sigma$ is a function of the superficial liquid Reynolds number (calculated from the liquid mass flow rate, the liquid kinematic viscosity, and the exit orifice diameter), and the ALR. Therefore, a least squares fit was performed over the range of superficial liquid Reynolds number and ALR for each spray

$$\sigma = K \cdot \text{Re}_{l}^{c} \cdot \text{ALR}^{d}, \tag{11}$$

with the superficial liquid Reynolds number defined as

$$\text{Re}_{l} = \frac{4 m_{l}}{\pi d_{o} \mu_{l}}. \tag{12}$$

The values are $K = 40.0$, $c = -0.37$, $d = 0.15$. The correlation coefficient obtained for this regression is 0.95. The value of $\sigma$ predicted by Eq. (10) is within 20% of the actual value for all the sprays investigated.

Recall that $\sigma$ is the scaling parameter for the ratio $r/x$ in the assumed form of the velocity profile given by Eqs. (1) and (2). Therefore, the value of $\sigma$ given by Eq. (11) is a measure of the spray cone angle. From our experiments, we have observed that the cone angle of effervescent sprays increases with an increase in ALR and decreases with an increase in the superficial Reynolds number. This fact is reflected in the values of $c$ and $d$ obtained for Eq. (11).

The fact that $b$ and $\sigma$ scale with ALR and superficial Reynolds number points out the fact that droplet velocity profiles in effervescent sprays are self-similar, but not universal. This is best understood from a study of cone angles. The cone angle of a turbulent gas jet is independent of Reynolds number in the fully turbulent regime. Therefore, turbulent gas jets exhibit universality. But as effervescent spray cone angles scale with ALR and superficial Reynolds number, they can at best be expected to exhibit self-similarity, not universality.

The error in the estimation of $b$ most likely arises from the estimation of $\sigma$. The parameters are directly related to each
other via continuity of the liquid phase. An under-estimation
of \( \sigma \) would result in an over estimation of \( b \), because a
smaller \( \sigma \) would mean that the same mass is now moving
through an area of lesser cross section, giving rise to higher
velocities.

Knowing \( b \) and \( \sigma \), one can use Eq. (1) to predict the
droplet velocity at any point in the dilute region of the
spray and under any operating conditions. Fig. 14 is a plot
of the normalized velocity versus \( r/x \) for a representative
spray. As can be seen, the spray exhibits self similarity in
the mean droplet velocity. The discrepancy between predic-
tions and experimental data could be because of either, or
both, of two factors. Firstly, there is a skewness in some of
the velocity profiles which arises from trying to identify the
centerline by differentiating the exact polynomial. As is
known, numerical differentiation is an error prone method;
this error could not be eliminated. Secondly, the spray may
not be fully developed. As a result, discrepancies between
model predictions and experimental data at axial positions
closer to the nozzle would be higher than corresponding
comparisons at positions farther away.

6. Summary and conclusions

Effervescent atomization was studied at high liquid mass
flow rates (30–120 g/s) and air-liquid-ratios by mass
(ALRs) ranging from 1% to 10%. The spray was character-
ized in terms of its radial velocity profiles at various axial
positions (all in the dilute spray, or far-field region), and in
terms of the variation in Sauter mean diameter (SMD)
across the spray.

The velocity profile was found to be bell shaped regard-
less of the ALR, liquid mass flow rate, or far-field axial
position downstream of the atomizer exit. The maximum
velocity occurred along the centerline of the spray. As
expected, the magnitudes of the droplet velocities increased
with an increase in either ALR or liquid mass flow rate. In
addition, the anticipated decrease in maximum velocity with
an increase in axial distance from the nozzle exit orifice was
noted.

The SMD data are at or below the target value of 75 \( \mu \)m
for the majority of locations throughout the spray. In addi-
tion, SMD is sensibly constant across any diameter of the

Fig. 12. Calculated centerline velocity versus the dimensionless group \( U \).

Fig. 13. Variation of \( b \) with air–to–liquid ratio by mass (ALR).

Fig. 14. Normalized mean velocity versus \( r/x \) for a 30 g/s, 10% ALR spray.
spray. This behavior is observed irrespective of liquid mass flow rate or ALR. It is ascribed to the high turbulence levels in the spray and the resulting good mixing. Further, drop size profiles do not show a significant change with either liquid mass flow rate or ALR.

Based on the experimental findings, and size-velocity-number flux plots provided by Panchagnula [25], we developed a model assuming variable-density single-phase flow to explain the dilute spray region droplet velocity profile and its variation with liquid mass flow rate and ALR. The model identified dimensionless groups in terms of the liquid momentum rate and entrained gas density to predict the far-field spatial variation in centerline velocity. The momentum rate in these sprays was measured using a momentum rate probe.

The dilute spray radial droplet velocity profiles show a variation with axial distance comparable to that of free gas jets, in that they are self-similar. However, the droplet velocity profile full width at half maximum (FWHM) is found to increase with an increase in ALR and to decrease with an increase in liquid mass flow rate. Thus, these velocity profiles are not universal.

In conclusion

1. Mean drop sizes of 75 μm and below were measured for most spray locations.
2. The variable density single-phase flow model was found to be appropriate for these sprays, based on experimental observations.
3. The velocity model provides a description of the spatial velocity distribution in effervescent sprays to within 30%.
4. Effervescent sprays have been found to have self-similar mean velocity profiles, but not universal.

Acknowledgements

We thank The Dow Chemical Company for sponsorship, and C. W. Lipp for numerous helpful discussions.

References